Multiple Beacon based Robust Cooperative Spectrum Sensing in MIMO Cognitive Radio Networks

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***Abstract*—This paper introduces novel detection schemes for multiple beacon signaling based cooperative spectrum sensing in multiple-input multiple-output (MIMO) wireless cognitive radio (CR) networks with channel state information (CSI) uncertainty. We consider a scenario in which the fusion center employs soft combining of the samples sensed by the cooperating secondary users corresponding to the primary user base-station beacon signals. We formulate the multiple beacon signaling based robust detectors, namely the robust estimator-correlator detector (RECD) and the robust generalized likelihood detector (RGLD), that can be employed at the fusion center towards primary user detection incorporating CSI uncertainty for cooperative spectrum sensing in MIMO CR networks. Further, we formulate a deflection coefficient based optimization framework and derive the optimal beacon sequence to maximize the probability of primary user detection at the fusion center. Simulation results show that the proposed robust detectors yield a significant improvement in the detection performance compared to the conventional CSI uncertainty agnostic matched filter (MF) detector for cooperative spectrum sensing in MIMO CR networks. Moreover, the optimal beacon signaling based robust detectors result in additional enhancement in the accuracy of primary user detection over suboptimal beacon structure based detectors.**

I. Introduction

The increase in the demand for higher data rates in modern broadband wireless networks has led to a scramble for the already scarce spectrum resources. This situation is further worsened by the inefficient bandwidth utilization arising from static spectrum allocation to the licensed/ primary users in traditional wireless networks. This progressive decrease in wireless bandwidth availability has inspired the development of cognitive radio (CR) technology [1]–[3]. CR systems are based on the principle of dynamically allocating vacant spectral bands, originally allocated to licensed/ primary users, to unlicensed/ secondary users, thereby improving the efficiency of spectrum utilization. Such vacant spectral bands, which are available intermittently during periods of primary user inactivity, are termed as *spectral holes* in CR networks. Hence, it is essential to reliably detect spectrum holes in cognitive scenarios to avoid causing interference to ongoing primary user radio transmission. This process of primary user detection based on radio channel measurements is termed as *spectrum sensing*.

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Several spectrum sensing schemes have been proposed in existing literature for primary user detection in cognitive radio networks. A comprehensive summary of these schemes can be found in works such as [4], [5]. It has been shown in literature [6] that cooperative spectrum sensing schemes, where the fusion center employs a soft combination of the sensed spectrum measurements from the cooperating secondary users, leads to a significantly enhanced accuracy of primary user detection in comparison to local sensing techniques, due to their resilience to wireless channel impairments such as fading, co-channel interference and hidden terminals. As shown in [7], the superior performance of soft-decision based cooperative detection schemes depends largely on the accuracy of the channel state information (CSI) at the fusion center. However, in practical scenarios one can only obtain CSI with limited accuracy due to error in the channel estimates and limited feedback on the reverse link. This process is additionally challenging in cooperative scenarios involving multiple secondary users.

Hence, in this context, we present robust detectors, which incorporate CSI uncertainty, for cooperative spectrum sensing in MIMO CR networks. Without loss of generality, we consider multiple primary user base-station beacon transmission based schemes for cooperative primary user detection. Towards this end, we develop the random signal model based robust estimator-correlator detector (RECD) followed by the unknown parameter model based robust generalized likelihood detector (RGLD) for cooperative spectrum sensing in the presence of CSI uncertainty. Moreover, the proposed robust sensing schemes consider different levels of uncertainty at the cooperating secondary users. Thus, the presented techniques are general in nature and can be readily employed in wireless scenarios with different fading channel conditions and estimation accuracies at the secondary users. Further, we formulate a deflection coefficient based bi-criterion optimization framework and derive the optimal beacon sequence towards maximization of the primary user detection probability. Simulation results demonstrate that the proposed robust detection techniques RECD, RGLD have a significantly superior detection performance in comparison to the uncertainty agnostic

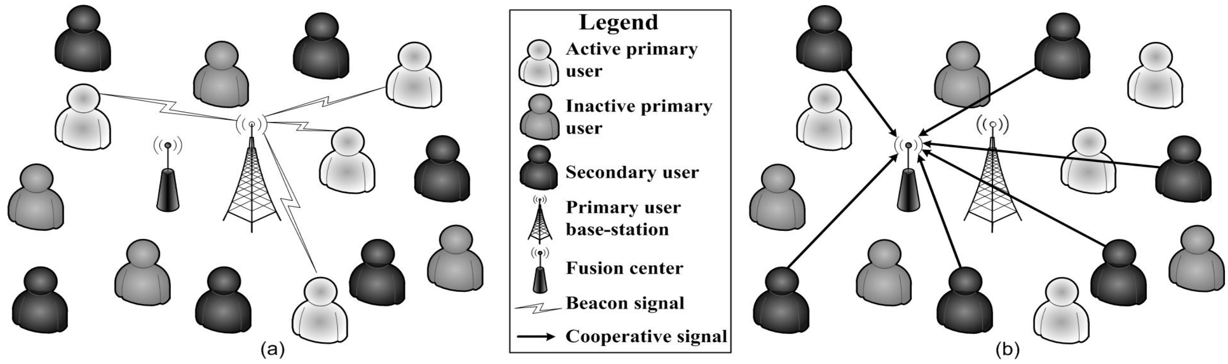


Fig. 1. (a) Cognitive Radio (CR) Network with active and inactive primary users and sensing secondary users, (b) Secondary users cooperate to convey the sensed data to the fusion center for primary user detection.

matched filter (MF) detector. This accuracy of detection is shown to be further enhanced by the employment of the optimal primary user base-station beacon sequence.

The paper is organized as follows. Section II describes the system model for MIMO CR networks with multiple secondary users followed by the description of uncertainty model considered for the channel estimates. In section III we formulate the proposed RGLD and RECD detectors for cooperative spectrum sensing scenario. Section IV describes a framework to construct the optimal beacon sequence for cooperative spectrum sensing scenarios. Further, simulation results are presented in section V and we conclude with sectionVI.

II. System Model

Consider a cooperative spectrum sensing scenario in a CR network with a primary user base-station, fusion center and N cooperating secondary users. Further, we assume a multi-user multiple-input multiple-output (MIMO) wireless CR network with N_t transmit antennas at the primary user base-station and N_r receive antennas at each of the cooperating secondary users. The baseband system model for the scenario described above at the n^{th} sampling instant is given as,

$$\mathbf{y}_i(n) = \mathbf{H}_i \mathbf{x}(n) + \mathbf{w}_i(n),$$

where $\mathbf{y}_i(n) \in \mathbb{C}^{N_r \times 1}$ is the received signal vector at the i^{th} secondary user corresponding to the primary user base-station broadcast beacon signal $\mathbf{x}(n) \in \mathbb{C}^{N_t \times 1}$ and the vector $\mathbf{w}_i(n) \in \mathbb{C}^{N_r \times 1}$ is the additive spatio-temporally white Gaussian noise at the i^{th} secondary user with covariance $\mathbf{R} = \mathrm{E}\left\{\mathbf{w}_i(n)\mathbf{w}_i^H(n)\right\} = \sigma^2 \mathbf{I}_{N_r}$. Each $\mathbf{H}_i \in \mathbb{C}^{N_r \times N_t}$, $1 \leq i \leq N$, is the MIMO wireless fading channel matrix, where the element $h_i(r,t)$ of the channel matrix denotes the flat-fading channel coefficient between the t^{th} transmit antenna of the primary user base-station and the r^{th} receive antenna of the i^{th} secondary user. The concatenated channel matrix $\mathbf{H} \in \mathbb{C}^{NN_r \times N_t}$ corresponding to the multi-user scenario with N cooperating secondary users can be obtained as,

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_N \end{bmatrix}.$$

We consider a scenario in which each secondary user senses the beacon signal broadcast by the primary user base-station and transmits the sensor observations to the fusion center. This is schematically shown in Fig.1. Upon receiving the measurements from each of the secondary users, the fusion center collectively processes the received data to detect the presence/ absence of the primary user. Hence, for the system described above, the stacked fusion center signal $\mathbf{y}(n) = \left[\mathbf{y}_1^T(n), \mathbf{y}_2^T(n), \ldots, \mathbf{y}_N^T(n)\right]^T \in \mathbb{C}^{NN_r \times 1}$ corresponding to the broadcast beacon signal $\mathbf{x}(n)$ can be described as,

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{w}(n), \tag{1}$$

where $\mathbf{w}(n) = \left[\mathbf{w}_1^T(n), \mathbf{w}_2^T(n), \ldots, \mathbf{w}_N^T(n)\right]^T \in \mathbb{C}^{NN_r \times 1}$ is the concatenated noise vector corresponding to N secondary users. In practical wireless scenarios it is significantly challenging to obtain accurate CSI due to the fast fading nature of the radio channel. Further, the channel estimates thus obtained are accurate only for a short coherence time interval due to the mobility of the wireless users. Hence, frequently, it is only possible to obtain a nominal channel estimate $\hat{\mathbf{H}}$ of the exact channel coefficient matrix $\mathbf{H}$. This uncertainty in the available channel estimate $\hat{\mathbf{H}}$ can be modeled as,

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{U}, \tag{2}$$

where the matrix $\mathbf{U} \in \mathbb{C}^{NN_r \times N_t}$ captures the uncertainty in the channel estimate arising due to the various factors listed above. The uncertainty matrix $\mathbf{U}$ can be modeled as $\mathbf{U} = \left[\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{NN_r}\right]^H$, where each row vector $\mathbf{u}_i^H \in \mathbb{C}^{1 \times N_t}$, $1 \leq i \leq NN_r$ follows a complex normal distribution, i.e. $\mathbf{u}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_\mathbf{u})$, with the uncertainty covariance matrix $\mathbf{R}_u = \mathrm{E}\left\{\mathbf{u}_i \mathbf{u}_i^H\right\}$. Using (2), the concatenated system model at the fusion center given in (1) can be equivalently formulated as,

$$\mathbf{y}(n) = \left(\hat{\mathbf{H}} + \mathbf{U}\right)\mathbf{x}(n) + \mathbf{w}(n).$$

In the above described system model at the fusion center for the cooperating secondary users, the signal $y_j^*(n)$ at the j^{th} receive antenna can be expressed as,

$$y_j^*(n) = \mathbf{x}(n)^H \left(\hat{\mathbf{h}}_j + \mathbf{u}_j\right) + w_j^*(n),$$

where $()^*$ denotes the complex conjugate and the vectors $\hat{\mathbf{h}}_j^H \in \mathbb{C}^{1 \times N_t}, 1 \leq j \leq NN_r$ form the rows of the concatenated channel estimate $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2, \ldots, \hat{\mathbf{h}}_{NN_H}]^H$. The concatenated fusion center signal at the j^{th} receive antenna corresponding to the L broadcast beacon vectors $\mathbf{x}(1), \mathbf{x}(2), \ldots, \mathbf{x}(L)$ can be equivalently represented as,

$$\mathbf{y}_j = \mathbf{X}\left(\hat{\mathbf{h}}_j + \mathbf{u}_j\right) + \mathbf{w}_j, \tag{3}$$

where the concatenated receive vector $\mathbf{y}_j = [y_j(1), y_j(2), \ldots, y_j(L)]^H \in \mathbb{C}^{L\times 1}$ is the signal at the j^{th} receive antenna corresponding to the beacon matrix $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \ldots, \mathbf{x}(L)]^H \in \mathbb{C}^{L\times N_t}$ obtained by concatenating the L beacon symbols. Similarly the concatenated noise vector $\mathbf{w}_j = [w_j(1), w_j(2), \ldots, w_j(L)]^H \in \mathbb{C}^{L\times 1}$ with covariance matrix $\mathbf{R}_w = \mathrm{E}\left\{\mathbf{w}_j\mathbf{w}_j^H\right\} = \sigma^2\mathbf{I}_L$. We consider a scenario in which the PU base-station transmits a beacon sequence $\mathbf{x}_k(n) \in \mathbb{C}^{N_t\times 1}$, for $1 \le n \le L$. The sequence of L beacon symbols are concatenated to form a beacon matrix $\mathbf{X}_k = [\mathbf{x}_k(1), \mathbf{x}_k(2), \ldots, \mathbf{x}_k(L)]^H \in \mathbb{C}^{L\times N_t}$ for $k = 0, 1$, indicating the absence, presence of the primary user signal respectively. It can be seen from (3) that the detection scenario at the j^{th} receive antenna for cooperative spectrum sensing can be equivalently formulated as the binary hypothesis testing problem,

$$\begin{aligned}\mathcal{H}_0 :\quad & \mathbf{y}_j = \mathbf{X}_0\left(\hat{\mathbf{h}}_j + \mathbf{u}_j\right) + \mathbf{w}_j\\ \mathcal{H}_1 :\quad & \mathbf{y}_j = \mathbf{X}_1\left(\hat{\mathbf{h}}_j + \mathbf{u}_j\right) + \mathbf{w}_j,\end{aligned} \tag{4}$$

where the null hypothesis $\mathcal{H}_0$ corresponds to the absence of the primary user signal and the alternative hypothesis $\mathcal{H}_1$ corresponds to the presence of the primary user signal. Next we describe novel detection techniques incorporating CSI uncertainty in cooperative spectrum sensing scenarios, namely the robust estimator-correlator detector (RECD) and the robust generalized likelihood detector (RGLD).

III. CSI Uncertainty Aware Robust Detection Schemes

A. Robust Estimator-Correlator Detection (RECD)

In this section we formulate the robust estimator-correlator detector [8] based on the statistical random signal model for the uncertainty in the CSI. The observed vector $\mathbf{y}_j$ described in (4), considering the transmission of the beacon matrices $\mathbf{X}_0 = \mathbf{0}_{L\times N_t}, \mathbf{X}_1$ corresponding to the absence and presence of the primary user signal respectively, follows a complex normal distribution described as,

$$\begin{aligned}\mathcal{H}_0 :\quad & \mathcal{CN}(\mathbf{0}, \mathbf{R}_w)\\ \mathcal{H}_1 :\quad & \mathcal{CN}\left(\mathbf{X}_1\hat{\mathbf{h}}_j, \mathbf{\Gamma}\right),\end{aligned}$$

where $\mathbf{\Gamma} = \mathbf{X}_1\mathbf{R}_u\mathbf{X}_1^H + \sigma^2\mathbf{I} \in \mathbb{C}^{L\times L}$ and $\mathbf{R}_u$ is the uncertainty covariance matrix defined previously. For the scenario described above the joint likelihood ratio corresponding to the concatenated observation matrix $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{NN_r}] \in \mathbb{C}^{L\times NN_r}$ for the N_r receive antennas at the N secondary users can be computed as,

$$\begin{aligned}L(\mathbf{Y}) &= \frac{\prod_{j=1}^{NN_r} p(\mathbf{y}_j; \mathcal{H}_1)}{\prod_{j=1}^{NN_r} p(\mathbf{y}_j; \mathcal{H}_0)},\\ &= \frac{\prod_{j=1}^{NN_r} \exp\left(-\left(\mathbf{y}_i - \mathbf{X}_1\hat{\mathbf{h}}\right)^H \mathbf{\Gamma}^{-1}\left(\mathbf{y}_i - \mathbf{X}_1\hat{\mathbf{h}}\right)\right)}{\prod_{j=1}^{NN_r} \exp\left(-\mathbf{y}_i^H\mathbf{R}_w^{-1}\mathbf{y}_i\right)}.\end{aligned}$$

Simplifying the joint likelihood ratio obtained above, the RECD test statistic for robust cooperative spectrum sensing incorporating uncertainty in the estimate of the channel matrix can be formulated as,

$$\begin{aligned}&T_{\text{RECD}}(\mathbf{Y}) =\\ &\sum_{j=1}^{NN_r}\left(\mathbf{y}_j^H\mathbf{\Gamma}^{-1}\mathbf{X}\hat{\mathbf{h}}_j + \frac{1}{2}\mathbf{y}_j^H\mathbf{R}_w^{-1}\mathbf{X}\mathbf{R}_u\mathbf{X}^H\mathbf{\Gamma}^{-1}\mathbf{y}_j\right).\end{aligned} \tag{5}$$

Based on the Neyman-Pearson criterion, the optimal detector that maximizes the probability of detection P_D for a given rate of false alarm P_{FA} is given as

$$T_{\text{RECD}}(\mathbf{Y}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma,$$

where we decide in favour of alternative hypothesis $\mathcal{H}_1$ and the null hypothesis $\mathcal{H}_0$ corresponding to the presence/ absence of the primary user depending on whether the obtained test statistic $T_{\text{RECD}}(\mathbf{Y})$ (5) is greater/ lesser than the threshold γ. The uncertainty aware robust estimator-correlator detector proposed above for cooperative spectrum sensing scenarios has a superior performance compared to the uncertainty agnostic matched filter detector because of the fact that the RECD has the ability to exploit the statistical information in the channel uncertainty.

B. Robust Generalized Likelihood Detection (RGLD)

In this section we employ the generalized likelihood ratio test (GLRT) framework to develop the RGLD test for cooperative spectrum sensing with CSI uncertainty. Consider the alternative hypothesis $\mathcal{H}_1$ corresponding to the presence of the primary user. Let the vector $\mathbf{r}_j$ be defined as $\mathbf{r}_j = \mathbf{y}_j - \mathbf{X}_1\hat{\mathbf{h}}_j$. Thus, the cooperative scenario described in (3) for the L concatenated sensed samples corresponding to the j^{th} receive vector of the total NN_r secondary user observation vectors can be equivalently derived as,

$$\begin{aligned}\mathbf{r}_j &= \mathbf{X}_1\mathbf{u}_j + \mathbf{w}_j,\\ &= \underbrace{\begin{bmatrix}\mathbf{X}_1 & \mathbf{I}_L\end{bmatrix}}_{\mathbf{A}}\underbrace{\begin{bmatrix}\mathbf{u}_j\\ \mathbf{w}_j\end{bmatrix}}_{\mathbf{z}_j},\end{aligned} \tag{6}$$

where $\mathbf{z}_j = \left[\mathbf{u}_j^T, \mathbf{w}_j^T\right]^T \in \mathbb{C}^{(L+N_t)\times 1}$ denotes the concatenated unknown random vector and $\mathbf{I}_L$ denotes the $L \times L$ identity matrix. The likelihood of the observation vector $\mathbf{y}_j$ parameterized by $\mathbf{z}_j$ corresponding to the alternative hypothesis $\mathcal{H}_1$ can be derived as,

$$p(\mathbf{y}_j; \mathbf{z}_j, \mathcal{H}_1) = \frac{1}{\pi^{L+N_t}|\mathbf{R}_z|}\exp\left(-\mathbf{z}_j^H\mathbf{R}_z^{-1}\mathbf{z}_j\right), \tag{7}$$

where $\mathbf{R}_z \in \mathbb{C}^{L+N_t \times L+N_t}$ denotes the covariance matrix $\mathbf{R}_z = \mathrm{E}\left\{\mathbf{z}_j \mathbf{z}_j^H\right\}$ of the random parameter $\mathbf{z}_j$ and is given as,

$$\mathbf{R}_z = \begin{bmatrix} \mathbf{R}_u & \mathbf{0}_{N_t \times L} \\ \mathbf{0}_{L \times N_t} & \mathbf{R}_w \end{bmatrix},$$

and $|\mathbf{R}_z|$ denotes the determinant of the matrix $\mathbf{R}_z$. Now, employing the GLRT framework, one can compute the estimate $\hat{\mathbf{z}}_j$, of the parameter vector $\mathbf{z}_j$ that maximizes the likelihood $p\left(\mathbf{y}_j; \mathbf{z}_j, \mathcal{H}_1\right)$ in (7). It can be readily seen that the problem of maximizing the likelihood can be equivalently formulated as the optimization problem,

$$\min.\ \mathbf{z}_j^H \mathbf{R}_z^{-1} \mathbf{z}_j\,, \quad \mathbf{r}_j = \mathbf{A}\mathbf{z}_j. \tag{8}$$

The estimate $\hat{\mathbf{z}}_j$ can be obtained as the solution of the standard weighted minimum norm optimization problem above (8) and is given as [9],

$$\hat{\mathbf{z}}_j = \mathbf{R}_z \mathbf{A}^H \left(\mathbf{A}\mathbf{R}_z\mathbf{A}^H\right)^{-1} \mathbf{r}_j.$$

The test statistic $T_{RGLD}\left(\mathbf{Y}\right)$ for the GLRT based RGLD for primary user detection in cooperative spectrum sensing scenarios is given as,

$$\begin{aligned} T_{\text{RGLD}}\left(\mathbf{Y}\right) &= \log\left(\frac{\prod_{j=1}^{NN_r} p\left(\mathbf{y}_j; \hat{\mathbf{z}}_j, \mathcal{H}_1\right)}{\prod_{j=1}^{NN_r} p\left(\mathbf{y}_j; \mathcal{H}_0\right)}\right), \\ &\doteq \sum_{j=1}^{NN_r} -\hat{\mathbf{z}}_j^H \mathbf{R}_z^{-1} \hat{\mathbf{z}}_j + \mathbf{y}_j^H \mathbf{R}_w^{-1} \mathbf{y}_j, \end{aligned} \tag{9}$$

where $\doteq$ denotes an equivalence up to an appropriate constant factor. It can be seen from the simulation results that the RGLD statistic based cooperative primary user detection yields a superior performance in comparison to the uncertainty agnostic matched filter detection. In the next section we formulate the optimization framework to derive the optimal beacon sequence $\mathbf{X}_1$, which further enhances the performance of the detection schemes presented above.

IV. Optimal Beacon Formulation

Let the vector $\tilde{\mathbf{r}} = \left[\mathbf{r}_1^T, \mathbf{r}_2^T, \ldots, \mathbf{r}_{NN_r}^T\right]^T$ be obtained by stacking the vectors $\mathbf{r}_j, 1 \leq j \leq NN_r$ corresponding to the total NN_r receive antennas in the cooperative spectrum sensing based CR network. Hence the system model described in (6) when concatenated for the total NN_r receive antennas corresponding to N secondary users is equivalently given as,

$$\tilde{\mathbf{r}} = \underbrace{\left(\mathbf{I}_{NN_r} \otimes \mathbf{X}_1\right)}_{\tilde{\mathbf{X}}} \mathrm{vec}\left(\mathbf{U}^H\right) + \tilde{\mathbf{w}},$$

where $\tilde{\mathbf{X}} = \left(\mathbf{I}_{NN_r} \otimes \mathbf{X}_1\right) \in \mathbb{C}^{LNN_r \times N_t NN_r}$ and the operator $\otimes$ denotes the matrix Kronecker product. The vector $\mathrm{vec}\left(\mathbf{U}^H\right)$ is the column vector that results from stacking the columns of the uncertainty matrix $\mathbf{U}^H$. The corresponding uncertainty covariance matrix $\mathbf{R}_U = \mathrm{E}\left\{\mathrm{vec}\left(\mathbf{U}^H\right)\left(\mathrm{vec}\left(\mathbf{U}^H\right)\right)^H\right\} = \mathbf{I}_{NN_r} \otimes \mathbf{R}_u$. Similarly, the concatenated noise vector $\tilde{\mathbf{w}} = \left[\mathbf{w}_1^T, \mathbf{w}_2^T, \ldots, \mathbf{w}_{NN_r}^T\right]^T$ with noise covariance matrix $\mathbf{R}_{\tilde{w}} = \mathrm{E}\left\{\tilde{\mathbf{w}}\tilde{\mathbf{w}}^H\right\} = \mathbf{I}_{NN_r} \otimes \mathbf{R}_w$. To determine the optimal beacon matrix $\mathbf{X}_1$ one needs to maximize the deflection coefficient $d^2\left(\mathbf{X}_1\right)$ for the primary user detection binary hypothesis testing problem [8]. This can be derived as,

$$\begin{aligned} d^2\left(\mathbf{X}_1\right) &\triangleq \frac{\left\|\mathrm{E}\left\{\mathbf{Y}; \mathcal{H}_1\right\} - \mathrm{E}\left\{\mathbf{Y}; \mathcal{H}_0\right\}\right\|_2^2}{\mathrm{tr}\left(\mathrm{cov}\{\mathbf{Y}; \mathcal{H}_1\}\right)}, \\ &= \frac{\left\|\tilde{\mathbf{X}}\ \mathrm{vec}\left(\hat{\mathbf{H}}^H\right)\right\|_2^2}{\mathrm{tr}\left(\tilde{\mathbf{X}}\mathbf{R}_U\tilde{\mathbf{X}}^H + \mathbf{R}_{\tilde{w}}\right)}, \\ &= \frac{1}{NN_r} \frac{\sum_{j=1}^{NN_r} \left\|\mathbf{X}_1 \hat{\mathbf{h}}_j\right\|_2^2}{\mathrm{tr}\left(\mathbf{X}_1\mathbf{R}_u\mathbf{X}_1^H + \mathbf{R}_w\right)}, \end{aligned}$$

where $\mathrm{E}\left\{\mathbf{Y}; \mathcal{H}_0\right\}$ and $\mathrm{E}\left\{\mathbf{Y}; \mathcal{H}_1\right\}$ denote the expected value of the observation vector $\mathbf{Y}$ under the null hypothesis $\mathcal{H}_0$ and alternative hypothesis $\mathcal{H}_1$ respectively and $\mathrm{cov}\left(\mathbf{Y}; \mathcal{H}_1\right)$ denotes the covariance of $\mathbf{Y}$ under the alternative hypothesis $\mathcal{H}_1$. Hence, the optimization framework for the optimal beacon matrix $\mathbf{X}_1$ that maximizes the performance of the detector for cooperative spectrum sensing scenarios, can be formulated as,

$$\begin{aligned} \max.\ & \sum_{j=1}^{NN_r} \left\|\mathbf{X}_1 \hat{\mathbf{h}}_j\right\|_2^2 - \lambda\left(NN_r\right)\ \mathrm{tr}\left(\mathbf{X}_1\mathbf{R}_u\mathbf{X}_1^H + \mathbf{R}_w\right) \\ \text{s.t.}\ & \mathrm{tr}\left(\mathbf{X}_1\mathbf{X}_1^H\right) \leq P_0, \end{aligned} \tag{10}$$

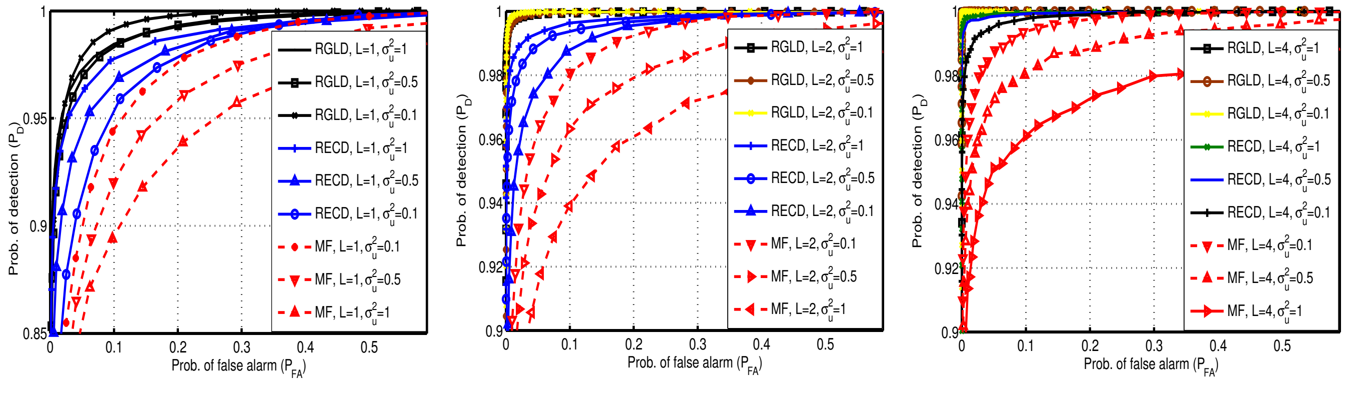
where λ is a non-negative constant and P_0 is the total transmit beacon power. The above optimization framework is a bi-criterion optimization problem where the choice of the constant λ allows a tradeoff between the uncertainty variance and the separation between the two hypothesis vectors. The optimization problem defined in (10) can equivalently be reduced to,

$$\begin{aligned} \max.\ & \mathrm{tr}\left(\mathbf{X}_1\mathbf{W}\mathbf{X}_1^H\right) \\ \text{s.t.}\ & \mathrm{tr}\left(\mathbf{X}_1\mathbf{X}_1^H\right) \leq P_0, \end{aligned} \tag{11}$$

where $\mathbf{W} = \sum_{j=1}^{NN_r}\left(\mathbf{h}_j\mathbf{h}_j^H\right) - \lambda NN_r\mathbf{R}_u$. The solution of the above optimization problem can be readily seen to be given by aligning each beacon vector $\mathbf{x}\left(i\right), 1 \leq i \leq L$, along the principal eigenvector of $\mathbf{W}$ corresponding to the largest eigenvalue. In the next section we present simulation results to validate the performance of the proposed detection schemes towards cooperative spectrum sensing based primary user detection in CR networks.

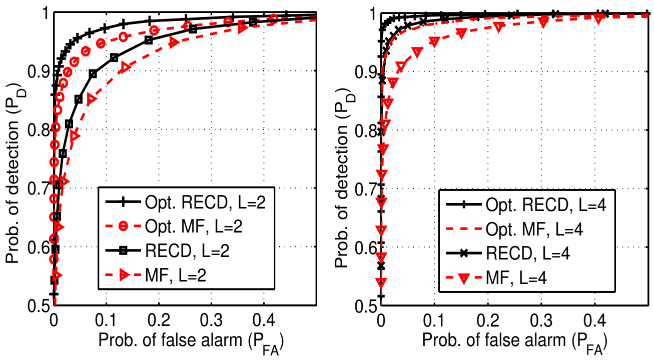
V. Simulation Results

We consider a wireless MIMO scenario in which the primary user base-station has $N_t = 2$ transmit antennas and each of the N secondary users has $N_r = 2$ receive antennas. The beacon signal vector corresponding to the null hypothesis $\mathcal{H}_0$, alternate hypothesis $\mathcal{H}_1$ are $\mathbf{x}_0\left(n\right)$, $\mathbf{x}_1\left(n\right)$ respectively for $1 \leq n \leq L$ where $\mathbf{x}_0\left(n\right) = \mathbf{0}_{2\times 1}$. Hence the beacon matrices $\mathbf{X}_0, \mathbf{X}_1 \in \mathbb{C}^{2\times L}$. Initially we choose the beacon matrix $\mathbf{X}_1$ as orthogonal. We model different levels of uncertainty by varying σ_u^2 in the uncertainty covariance matrix $\mathbf{R}_u = \sigma_u^2\mathcal{D}\left(\left[1, 0.9\right]\right)$, where $\mathcal{D}\left(\mathbf{a}\right)$ denotes the diagonal matrix



(a) (b) (c)

Fig. 2. Comparison between the robust generalized likelihood detector (RGLD), robust estimator-correlator detector (RECD) and matched filter (MF) for $N_r = 2$, $N_t = 2$ MIMO, $N = 2$, SNR=1dB, $\mathbf{R}_u = \sigma_u^2 \mathcal{D}(1, 0.9)$, (a) $L = 1$, (b) $L = 2$ and (c) $L = 4$.



(a) (b)

Fig. 3. Comparison of detection performance with optimal beacon signalling versus conventional beacon signaling for the robust estimator-correlator detector (RECD) and matched filter (MF) for $N_r = 2$, $N_t = 2$ MIMO, $N = 2$, SNR=-5dB, $\mathbf{R}_u = \mathcal{D}(1, 0.9)$ (a) $L = 2$ and (b) $L = 4$.

with the elements of vector $\mathbf{a}$ along its principal diagonal. Fig. 2(a)., Fig. 2(b). and Fig. 2(c)., show the probability of primary user detection (P_D) versus the probability of false alarm (P_{FA}) for the uncertainty aware robust estimator-correlator detector (RECD) in (5), robust generalized likelihood detector (RGLD) in (9) and the conventional matched filter (MF) detector with $L = 1, 2, 4$, beacon symbols, respectively. In each figure we consider different levels of uncertainty by varying $\sigma_u^2 \in \{0.1, 0.5, 1\}$. It is evident from each figure that the proposed robust detectors have a superior performance to the MF detector and it can also be seen that the RGLD has a performance edge over the RECD. Further, the performance gap between the robust detectors and the MF detector widens with increasing CSI uncertainty. Comparing the plots for increasing L in figures 2(a), 2(b) and 2(c), it is evident that the RECD and RGLD performances improves with increasing number of beacons vectors. Fig. 3. compares the detection performance of the optimal beacon signaling (11) proposed in section IV with the conventional orthogonal beacon signaling for the RECD and MF detectors. It is seen from the results that employing the optimal beacon sequence significantly boosts the performance of the detectors in comparison to the conventional beacon matrix.

VI. Conclusion

In this paper we presented multiple beacon signaling based RECD and RGLD detection schemes for cooperative spectrum sensing in MIMO CR networks, considering uncertainty in the CSI. Further we formulated a novel deflection coefficient based optimization framework to derive the optimal beacon matrix to maximize the performance of cooperative spectrum sensing based primary user detection. Simulation results have been presented to demonstrate the superior performance of the proposed multiple beacon signaling based uncertainty aware robust detection schemes, namely RECD and RGLD when compared with the MF detector. It has also been shown that the optimal beacon matrix further enhances the performance of the proposed detectors.

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